An Extended Passive Motion Paradigm for Human-like Posture & Movement Planning in Redundant Manipulators

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***Abstract*— A major challenge in robotics and computational neuroscience is relative to the posture/movement problem in presence of kinematic redundancy. We recently addressed this issue using a principled approach which, in conjunction with nonlinear inverse optimization, allowed capturing postural synergies such as Donders' law [1]. In this work, after presenting this general model specifying it as an extension of the Passive Motion Paradigm, we show how, once fitted to capture experimental postural strategies, the model is actually able to also predict movements. More specifically, the Passive Motion Paradigm embeds two main intrinsic components: joint damping and joint stiffness. In previous work we showed that joint stiffness is responsible for static postures and, in this sense, its parameters are regressed to fit to experimental postural strategies. Here, we show how joint damping, in particular its anisotropy, directly affects task-space movements. Rather than using damping parameters to fit *a posteriori* task-space motions, we make the *a priori* hypothesis that damping is proportional to stiffness. This remarkably allows a postural-fitted model to also capture dynamic performance such as curvature and hysteresis of task-space trajectories during pointing tasks, confirming and extending previous findings in literature.**

I. Introduction

Recent trends in both industry and healthcare clearly show the need for robots to be able to cooperate and assist humans in specific tasks. In order to do so, not only our robots will need to be safe-by-design, incorporating for example compliant mechanisms [2] and force/impedance control architectures [3] (as opposed to the current rigid and position-controlled deployed in industry) but will also need to *behave* naturally. In other words, while working with a robot, human operators not only need to be safe at all times, but shall also feel comfortable. As an example, imagine a robotic assistant designed to hand-over tools to a human operator. It is quite important for the robot to assume *natural postures*, which carry non-verbal semantics very valuable to human operators (the same object can be passed in different ways, for different purposes). For this and other reasons, in the last decades, roboticists have started looking into human motor strategies as a source of inspiration for the formulation of bio-inspired postural/motion controllers [4], [5], [6], [7].

The coordination of redundant degrees-of-freedom is a central topic in both robotics and neuroscience and we are interested in two specific aspects: the *redundancy problem* [8] and the *posture/movement problem* [9]. This issue was first addressed by the authors in a recent work [1] where a principled approach was proposed to tackle these very issues with focus on capturing postural strategies. In this work, we shall specialize the computational model and shift the focus on movement generation rather than joint-space postural strategies.

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Computational approaches to posture and movement

Despite in the presence of kinematic redundancy there are infinite postures for the same end-effector position and orientation, an extensive number of behavioural studies showed that, during kinematically redundant tasks, humans adopt a stereotypical strategy (also known as Donder's Law) that associates a *unique* and *path-independent* posture to a given task (see [10], [1] and references therein). It has been suggested that the brain implements postural synergies (such as Donders' Law) as a flexible family of *holonomic constraints* [11], [12] to solve redundancy (i.e. by *reducing* the number of degrees-of-freedomg effectively availble to perform a given task) as well as to fulfill some *optimality* criteria that might vary in different experimental scenarios and physiological conditions [13], [11], [14]. From a computational perspective, Donders-like postural strategies can be captured by solving a constrained optimization problem which returns the unique optimal posture that minimizes a given (posture-dependent) objective function while fulfilling a desired task-constraint [15], [16], [17], [10]. Because these *postural models* only compute static/equilibrium-configurations they are usually not suitable for planning movements.

Postures are somewhat *static*, possibly accounted for as equilibria of some potential field [17], while movement is in apparent contrast with the very concept of equilibrium. The Posture/Movement problem stems out from the possible interference of postural control mechanisms with general motor strategies [9]. In the last few decades, various approaches have been proposed in computational neuroscience as an attempt to reconcile posture and movement.

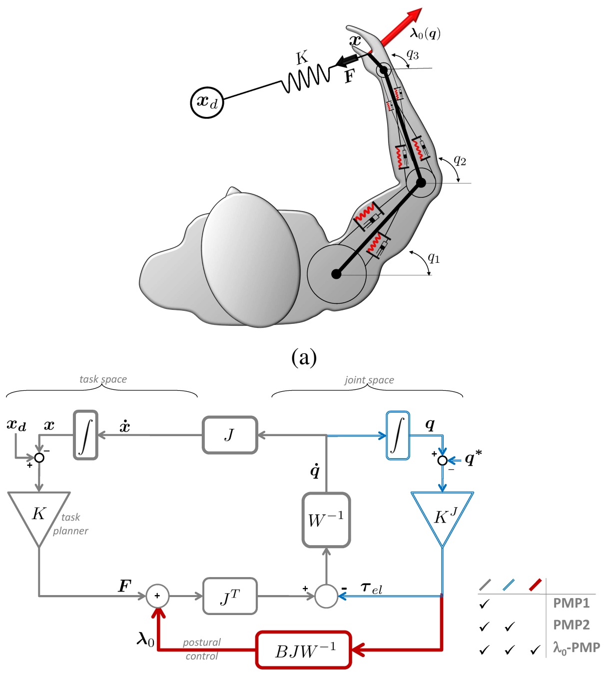
Transport models [18] such such as minimum-torque-change [19], minimum-work [20], minimum-variance [18], do provide a solution to the Posture/Movement problem but, in their original formulation are *incompatible* with Donders' law as they predict path-dependent equilibrium postures [21].

Another approach is the so-called Passive Motion Paradigm (PMP) [22], [23]. PMP can be considered a computational generalization of the Equilibrium Point Hypothesis [24], in that *goals and kinematic constraints can be superimposed when viewed as force-fields*. First proposed in the 80s [22], the PMP has evolved over the years and has been applied as a framework for human motor control as well as to motion planning for humanoid robots [25], [23]. One of the major strengths of the PMP lies in its computational simplicity. While full details of the PMP are found in [23] and references therein, its basic features are illustrated in Fig.1. In the standard PMP model, a robotic manipulator is seen as a rigid structure (i.e. an arm-like kinematic chain) with 'intrinsic' properties defined at the level of joint-space (e.g. joint angles q_1, q_2, q_3) and 'extrinsic' properties defined at the level of task-space (e.g. actual and desired endpoint postures $\boldsymbol{x}$ and $\boldsymbol{x}_d$, respectively). The redundancy problem is solved by postural mechanisms implemented via the action of an *intrinsic impedance*, for example in the form of purely viscous (mechanical dampers) or viscoelastic (dampers and springs) elements interconnected at joint level. On the other hand, movement is planned at task-level and implemented by an *extrinsic impedance*, $\boldsymbol{F} = K(\boldsymbol{x}_d - \boldsymbol{x})$ in Fig.1, acting as a generalized spring which continuously drives the end-effector (at some position $\boldsymbol{x}$) towards the goal $\boldsymbol{x}_d$ while the intrinsic impedance takes care of postures.

The standard PMP comes in two forms, with the only difference in terms of intrinsic impedance: one being *purely viscous* (PMP1 in Fig.1b) and the other being *viscoelastic* (PMP2 in Fig.1b). In the first case, it can be easily shown [1] that a purely viscous intrinsic impedance solves the posture/movement problem but is *incompatible* with Donders' law (as it does not yield repeatable postures). In the second case, the intrinsic viscoelastic impedance ensures unique postures (due to an elastic potential in joint-space) but does not solve the posture/movement problem as the intrinsic elastic torques ($\boldsymbol{\tau}_{el}$) 'pulls' the end-effector back to a rest position ($\boldsymbol{q}^*$), in contrast with the extrinsic spring which pulls the end-effector towards a target $\boldsymbol{x}_d$ [1].

Task-space Separation Principle and the extended Passive Motion Paradigm ($\boldsymbol{\lambda}_0$-PMP)

One way to prevent the interference between intrinsic and extrinsic elastic potentials is to *block any effect of the intrinsic potential onto the task*. Inspired by the *Separation Principle* (of static and dynamic torques) that has been derived from neuroscientific evidence [26], we recently proposed the $\boldsymbol{\lambda}_0$-PMP model [1], an extension of the standard PMP. In literature, the Separation Principle is typically applied at joint-space level (larger than task-space, dimension-wise, when dealing with redundant manipulators), assuming that static contributions (either due to gravity or to elastic fields) are perfectly compensated for by the brain (or by the robot controller) so that they can be removed from the dynamic equations of the limb under control [26], [27]. In our recent work [1], we derived the $\boldsymbol{\lambda}_0$-PMP model by $i)$ re-framing the redundancy problem within the *constrained minimization framework* and applying the Lagrange Multipliers method; $ii)$ noting that the Lagrange Multipliers $\boldsymbol{\lambda}$ define a *task-space force field*; $iii)$ applying the Separation Principle to $\boldsymbol{\lambda}$ and defining a *static task-space force field* $\boldsymbol{\lambda}_0$ (from which the name of the method). This task-space force $\boldsymbol{\lambda}_0$, also highlighted in Fig.1 as an addition to the standard PMP, produces *partial compensation* of the elastic joint torques ($\boldsymbol{\tau}_{el}$), blocking their effect *only* on the task-space, leaving joint torques free to act in the *null-space*, and driving the posture towards minima of intrinsic potentials without interfering with task-space objectives. This captures the essence of postural synergies such as Donders' law which can now be seen as generated from a joint-space potential combined with a task-space force field.



(a)

(b)

Fig. 1: $\boldsymbol{\lambda}_0$-PMP as an extension of the Passive Motion Paradigm. (a) Example of a 3DOF human-like arm performing a redundant 2D reaching task. (b) Block diagram of the general $\boldsymbol{\lambda}_0$-PMP model.

With reference to Fig.1.(b), for the $\boldsymbol{\lambda}_0$-PMP, once the Jacobian matrix J is defined (given the geometry of the manipulator and of the task), the parameters which need to be determined are the *intrinsic stiffness matrix* K^J; the *intrinsic damping matrix* W (or, equivalently, the admittance W^{-1}); and the *extrinsic stiffness matrix* K.

In previous works, we focused on postural strategies and showed how Donders' Law can be captured via an intrinsic elastic potential [17], [10] and how nonlinear inverse optimization can be used to determine the coefficients of the intrinsic stiffness K^J to fit experimental postural synergies [1], [10]. In this work, we shift our focus on movement strategies, which, in our framework, are primarily shaped by the damping matrix W. Rather than trying to use the coefficients of the matrix W as 'extra degrees of freedom' to better fit experimental data *a posteriori*, we assume *a priori* that *damping is proportional to stiffness*, in line with experimental evidence [28], [29]. In other words, we hypothesize that the same biomechanical factors which determine the 'shape' of

K^J (i.e. its eigenvectors and eigenvalues) also determine the 'shape' of W.

With this hypothesis in place, the intrinsic stiffness still has an 'indirect effect' as it shapes the intrinsic damping matrix W, whose dynamic effects are not blocked by $\boldsymbol{\lambda}_0$. We shall specifically show how this mechanism determines curvature of task-space trajectories during pointing tasks performed with the wrist, in line with the experimental evidence also reported in literature by Charles et al. [30].

II. Materials and Methods

This section presents the $\boldsymbol{\lambda}_0$-PMP, a novel extension of the Passive Motion Paradigm, and its specialization to wrist pointing tasks which will be later used in a comparative analysis with experimental data.

A. Donders-fitted $\boldsymbol{\lambda}_0$-PMP model for wrist-pointing tasks

Building on previous experimental and computational studies [31], [32], [17], [1], [33], [34], we are specifically interested in capturing human-like motor strategies during pointing tasks performed with the wrist. To implement the model in Fig.1-b, we shall first determine the Jacobian J from the forward kinematics; the intrinsic stiffness (K^J) matrix and the rest posture $\boldsymbol{q}^*$; the damping (W) matrix as well as the extrinsic stiffness matrix K.

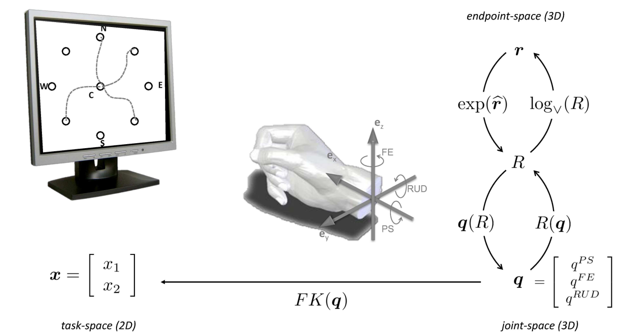


Fig. 2: Center-out task and kinematic spaces involved in wrist pointing. 3D wrist configurations can be equivalently expressed in terms of 3×3 rotation matrices $R(\boldsymbol{q})$, rotation vectors $\boldsymbol{r}$ or joint rotations $\boldsymbol{q}$. The forward kinematics $\boldsymbol{x} = FK(\boldsymbol{q})$ maps 3D wrist orientations onto the 2D task, expressed in screen coordinates $\boldsymbol{x}$. Adapted from [17].

Forward Kinematics: With reference to Fig.2, we assume that the wrist is used to point a virtual laser beam onto a point on a computer screen, i.e. a two-dimensional task-space of coordinates $\boldsymbol{x} = [x_1\ x_2]^T \in \mathbb{R}^2$. The human wrist is modelled as an ideal, three-dimensional gimbal comprising the following three orthogonal, rotational axes (from proximal to distal): The joint space is therefore three-dimensional and can be described via a joint vector $\boldsymbol{q} = [q^{PS}\ q^{FE}\ q^{RUD}]^T \in \mathbb{R}^3$ or, alternatively, via rotation vectors [32], [17], as shown in Fig.2.

The forward kinematics (FK) for a 3DOF wrist at distance $d = 1$ m and initially pointing in the $[1\ 0\ 0]^T$ direction can be written as

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = FK(\boldsymbol{q}) = \underbrace{\begin{bmatrix} 0 & -d & 0 \\ 0 & 0 & d \end{bmatrix}}_{\texttt{screen projection}} \cdot R(\boldsymbol{q}) \cdot \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad (1)$$

where $R(\boldsymbol{q})$ represents the 3D hand orientation, computed as $R(\boldsymbol{q}) = \exp(-\hat{\boldsymbol{e}}_x q^{PS}) \exp(\hat{\boldsymbol{e}}_z q^{FE}) \exp(\hat{\boldsymbol{e}}_y q^{RUD})$ where the exponential notation $\exp(\hat{\boldsymbol{e}}\,\theta)$ represents the rotation about an axis $\boldsymbol{e}$ by and angle θ [35]. Further details are given in [17] and references therein. Once the forward kinematics is given, the Jacobian can be analytically computed as: $J(\boldsymbol{q}) = \frac{\partial FK(\boldsymbol{q})}{\partial \boldsymbol{q}}$.

Subject-specific intrinsic stiffness K^J and rest posture $\boldsymbol{q}^*$ from experimental data: For the 3DOF wrist, the intrinsic stiffness (as well as the damping) is represented by a 3×3 *symmetric matrix*. The rest posture $\boldsymbol{q}^*$ represents the posture (three joint angles) of minimum elastic energy. Using nonlinear inverse optimization (NIO) techniques [1], a subject-specific matrix K^J and rest posture $\boldsymbol{q}^*$ can be directly derived from experimental data. As detailed in [1], experimental data consisting of thousands of data points are fitted to a quadratic surface, typically used in literature to encode Donders' law. This can be seen as an extreme downsampling of experimental data and the resultant quadratic surface can be seen as an average Donders' surface. The reader is referred to [1] for the detailed procedure based on nonlinear inverse optimization. One thing to highlight is that it is the relative *ratio* between eigenvalues of K^J which determines a specific Donders' law, not the absolute values. For this reason, the trace of the matrix can be set to any arbitrary (positive) number. To be in line with biomechanical (passive) stiffness values found in literature [36], we set this value to be $trace(K^J) = 4$ Nm/rad.

Damping W and intrinsic time constant: While the intrinsic stiffness matrix is derived directly from a fitting process of experimental data, for the intrinsic damping matrix W, rather than trying to use the coefficients of the matrix W as 'extra degrees of freedom' to better fit experimental data, we assume that *damping is proportional to stiffness*, in line with experimental evidence [28], [29], [37], [36]. In other words, we hypothesize that the same biomechanical factors which determine the 'shape' (in terms of eigenvectors and eigenvalues) of K^J will determine a similar 'shape' for W. For this reason we set the damping to be proportional to the intrinsic stiffness

$$W = \tau_0 K^J \quad (2)$$

where τ_0 is a scalar (positive) value with the units of time, and can be therefore thought of as an *intrinsic time constant*. The reason is that, for a simple scalar, linear spring-damper system, the ratio between damping and stiffness determines exactly the time constant of the system.

Task-space impedance and dynamics: With reference to Fig. 1b, the intrinsic friction W also affects task-space dynamics through the task-space damping $B(\boldsymbol{q})$ (see [1] for more details):

$$B(\boldsymbol{q}) = \left(JW^{-1}J^T\right)^{-1} \tag{3}$$

that transforms task-space velocities $\dot{\boldsymbol{x}}$ into task-space forces which balance out the effect of the extrinsic spring K, leading to the following task-space dynamic equation:

$$B(\boldsymbol{q})\dot{\boldsymbol{x}} = K(\boldsymbol{x}_d - \boldsymbol{x}) \tag{4}$$

Although the task-space damping is posture-dependent and more equations are needed to fully solve the dynamics, some remarkable properties can already be noted: $i)$ the task-damping $B(\boldsymbol{q})$ in (3) directly depends on the intrinsic damping W which, therefore, directly affects task-space dynamics (4); $ii)$ the intrinsic stiffness K^J does not appear in (4) (being the intrinsic elastic torques compensated in task-space by the $\boldsymbol{\lambda}_0$ force field) and therefore does *not* directly affect the task dynamics, however, it does it *indirectly* through postures adjustments in the null-space which affect the Jacobian and therefore $B(\boldsymbol{q})$.

The extrinsic stiffness K is responsible for the task-space dynamics together with the task-space damping $B(\boldsymbol{q})$, as highlighted in (4). However, $B(\boldsymbol{q})$ is determined once J and W are given, as in (3). In this work, we are considering very simple center-out tasks, as described below. As there is no *a priory* preferential direction in the task-space, we shall consider an *isotropic* extrinsic stiffness. Furthermore, to plan bell-shaped task velocities, following [38], we shall consider time-varying (isotropic) stiffness

$$K(t) = k \cdot \left(1 - e^{-\frac{t}{\tau}} - \frac{t}{\tau}e^{-\frac{t}{\tau}}\right) \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{5}$$

with the property of increasing its stiffness value from zero to k with an *extrinsic time constant* τ (notice that when pointing to a new target the time t is reset to zero).

III. Results

In this section we compare the average (of ten pointing trials) experimental pointing strategy[1] of a representative subject with the task and joint-space trajectories predicted by the Donders-fitted $\boldsymbol{\lambda}_0$-PMP model, i.e. whose intrinsic potential parameters K^J and $\boldsymbol{q}^*$ have been estimated via NIO from the experimental data as detailed in [1]. Note that these parameters alone only capture Donders' law, i.e. they can identify optimal postures for giving pointing directions [17], [10], [1] but cannot tell where to point. The actual motion, in particular the geometry of task-space trajectories, will be shaped by the task dynamics (4). Here, rather than fitting every single movement *a posteriori* with a specific damping W, we make the *a priori* hypothesis that *intrinsic damping is proportional to intrinsic stiffness*, via an intrinsic time constant as in (2).

As shown in (4), task-space dynamics depend on K and on $B(\boldsymbol{q})$ which, in turn, depends on the intrinsic damping W via (3). For the task-planner, we shall assume as extrinsic $K(t)$ as in (5), in particular *isotropic* and therefore *not directly responsible for path curvatures*. Both the intrinsic (τ_0) and extrinsic (τ) time constants are set on a movement-specific basis as $\tau = \tau_0 = \frac{T}{5}$, where T is the average time that the subject requires to perform a specific movement. Similar to (2), we used the time-constant τ_0 to tune the scalar stiffness k in (5) as:

$$k = b_{max}/\tau_0 \tag{6}$$

where b_{max} is the maximum eigenvalue of the matrix $B(\boldsymbol{q}_0)$ and $\boldsymbol{q}_0$ is the initial wrist configuration prior to the starting of the movement.

Fig. 3a compares the average experimental task-space paths with those predicted by the $\boldsymbol{\lambda}_0$-PMP model. We performed a curvature analysis as in [30] to elucidate if task-space paths are curved (thick lines in Fig.3a) and whether outbound and inbound movements follow different paths (stars in Fig.3a). A paired ttest ($p < 0.05$) between simulated and experimental task-space paths revealed that, for all movements, the differences between simulated and experimental paths were not statistically significative.

The experimental and simulated joint space trajectories are shown in Fig.3b (only outbound and inbound movements from the (E) the (W) target are shown). The subject show high variability when coordinating the PS rotation (red area), most likely because this is the joint that adds redundancy to the pointing task. The model can accurately reproduce the average FE (magenta color) and RUD (green) trajectories for most of the movements, while for PS rotations, there are larger errors between the average experimental trajectory and the model.

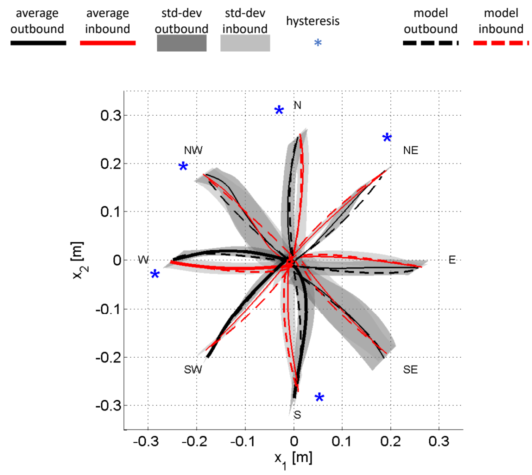
Fig. 3c compares the experimental task-space tangential velocity profile and those predicted by the model. The time-varying extrinsic spring (5) reproduces bell-shaped velocity profiles similar to the experimental ones, although, task-space velocities predicted by the model tend to be larger than the experimental ones.

IV. Conclusion

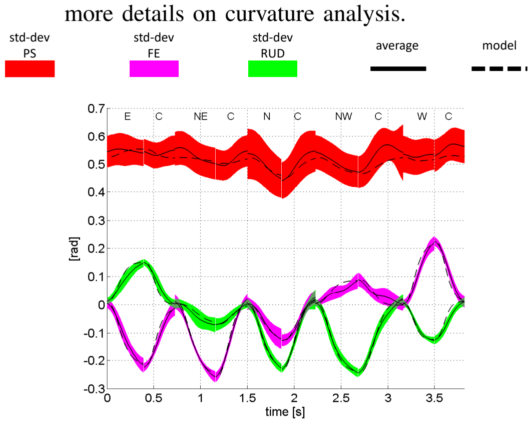
In this work we focused on human motor strategies during redundant pointing tasks performed with wrist (and forearm) rotations. In a previous work, Charles and Hogan [30] showed that when pointing with the wrist, task-space paths are curved and in general, inbound and outbound movements follow different paths. In a successive work, they posited that such features of wrist rotations are due to an anisotropic joint stiffness matrix.

Here, based on a novel model proposed in [1] devised to tackle the posture/movement problem, we put forward the hypothesis that anisotropic intrinsic damping, rather than stiffness, is primarily responsible for curved task-space paths. The novel aspect of our approach is that our model was fitted to capture postural strategies and, with the sole hypothesis that intrinsic damping is proportional to stiffness (2), the model also exhibited curvatures and hysteresis in task-space performance remarkably similar to subject-specific average motions. Due to space constraints, in this paper we only

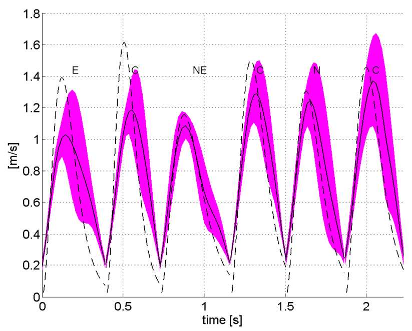
[1]The experimental protocol used to asses the experimental pointing strategy can be found in [31].



(a) Experimental vs simulated task-space paths. Thick lines mark curved movements ($p < 0.05$) while stars mark hysteretic paths, i.e. featuring different outbound and inbound paths. See [30] for more details on curvature analysis.



(b) Experimental vs simulated joint-space paths (only the first five outbound and inbound movements are shown).



(c) experimental vs simulated (dashed line) tangential velocity profiles (only the first three outbound and inbound movements are shown).

Fig. 3: Average experimental pointing strategies vs Donders-fitted λ_0-PMP model.

showed the results for a representative subjects and a more accurate and detailed analysis involving several subjects will be presented in a separate work.

There are of course many approximations and assumptions in our model which, as mentioned, is not meant to predict exact trajectories but rather capturing some basic features of human-like motion. A major limitation is that the intrinsic stiffness K^J is only a very simplified attempt to approximate the real, nonlinear, time-variant mechanical stiffness typically of human arm. This in turn affects not only the predicted postural strategies (i.e. wrist configuration at the beginning and ending of a movement) but also the predicted trajectories as the relationship between damping and stiffness is certainly more complex than the simple proportionality assumed in (2).

A second limitation is that the λ_0-PMP totally neglects feedback, as it is meant to address *motion planning* rather than execution. Our model is however useful at a planning stage, while feedback should be incorporated for movement execution.

As a third limitation, our model is to be considered as a *first order postural and motor planner*, in the sense that it does not take into account the inertial properties of the human or robotic arms. This is a specific choice (in some cases an inertia might not even be available, e.g. in motor imagery scenarios [23]) and the model could be extended to include inertial properties. In fact, the role that the manipulator intrinsic inertia would have is the same that the intrinsic damping has in our model. Such an approach would lead to models along the lines proposed by Khatib [39].

In conclusion this work presents an extended version of the PMP that can deal with kinematic redundancy in compliance with Donders' law and solve the posture/movement problem. Just like the PMP, our model can find extensive use in planning human-like motions for humanoid robots and, at the same time, be able to capture *natural postures* in compliance with Donders' Law.

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